KALMAN FILTER CHARACTERIZATION OF CESIUM CLOCKS AND HYDROGEN MASERS

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Abstract

Our previous PTTI paper demonstrated that a two-state Kalman filter involving the parameters frequency and frequency drift, when properly implemented, produces frequencies for hydrogen masers that generate a stable mean timescale, at least for cases involving simulated and low-noise real data. The current paper extends the investigation to the noisier data actually employed in the USNO operational timescale. Results for about 500 days of postprocessed data for 12 masers and 51 cesium clocks indicate comparable frequency stabilities to those obtained with the currently operational timescale algorithm.

1 INTRODUCTION

Mean timescales based on Kalman filter modelling of individual clocks have been investigated by others in order to provide an ensemble time that is more uniform than the time kept by any one of the constituent clocks and whose estimated states are optimal in the minimum squared error sense. Tryon and Jones [1], Barnes and Allan [2], Stein [3], and Greenhall [4] employed as state parameters the phase, frequency, and frequency drift of an ensemble of cesium clocks relative to a master clock. A problem with this type of filter is that, since time is unobservable, elements of the covariance matrix grow without bound [5]. Weiss and Weissert [5] utilized a one-state filter to determine the frequency of cesium clocks used in their AT2 timescale. Only Tryon and Jones and Weiss and Weissert tested their algorithms on real data.

Barnes and Allan [2] proposed a Kalman filter that utilized only frequency and drift, pointing out that only these two parameters are physically meaningful for a frequency standard, and indeed showed by simulations that such a two-parameter filter performed better than the three-parameter one. Since the phase (time) of a clock is integrated from the frequency, it adds no information, and solution for a phase parameter is unnecessary.

Consequently, a reasonable model for both our cesium-beam frequency standards and our hydrogen masers (which generally have inherent frequency drift) is:

$$f(t + \Delta t) = f(t) + \Delta t \cdot d(t) + \varepsilon(t + \Delta t)$$

$$d(t + \Delta t) = \qquad\qquad d(t) + \eta(t + \Delta t)$$

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14. ABSTRACT
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where $f(t)$ is a clock's frequency at time $t = 1, 2, \ldots$; $d(t)$ is its frequency drift; $\varepsilon(t)$ and $\eta(t)$ are independent random variables with zero mean and normal distributions uncorrelated in time (zero autocorrelation), i.e. white noise processes; and Δt is the time step. The latter is 1 hour in our case, which is in the white FM noise regime of cesiums and in the flicker FM regime of masers. For the masers, measurement system noise dominates clock noise for sampling times up to a few hours and is characterized by white phase noise. A Kalman filter that assumes the presence of only white FM and random-walk FM noises cannot exactly model 1-hour data from a maser. However, in our previous paper **[6]**, it was shown that, for an ensemble of hydrogen masers connected to a low-noise (Timing Solutions Corporation or TSC) measurement system, such a filter with the parameters of frequency and frequency drift is quite capable of producing a mean timescale with a frequency stability that is not only viable, but at sampling times up to several days, actually superior to that produced by a conventional least-squares algorithm such as the currently operational at USNO **[7]**. Among other issues, this paper addresses whether a viable timescale can be similarly generated for the noisier data from our currently operational Data Acquisition System.

In order to minimize process noise, one must choose a reference that is stable as possible. The Mean timescale, at least as derived from the clocks under consideration, cannot so serve because it is not yet available at the first step of a recursion. One could refer all the frequencies and drifts to one of the clocks or to a Mean predicted from previous data. However, it would be easier and more error- and correlation-free to refer one type of clocks, say the masers, to the Mean of the other type of clocks (cesiums); this is our objective. In USNO's operational timescale, the maser rates and drifts are calibrated against the cesium Mean anyway, which is always available.

2 THE KALMAN FILTER

The state transition equations are:

$$
\begin{bmatrix} f_1(t+\Delta t) \\ d_1(t+\Delta t) \\ f_2(t+\Delta t) \\ d_2(t+\Delta t) \\ \vdots \end{bmatrix}
=
\begin{bmatrix}
1 & \Delta t & 0 & 0 & \cdots \\
0 & 1 & 0 & 0 & \cdots \\
0 & 0 & 1 & \Delta t & \cdots \\
0 & 0 & 0 & 1 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix}
\begin{bmatrix} f_1(t) \\ d_1(t) \\ f_2(t) \\ d_2(t) \\ \vdots \end{bmatrix}
+
\begin{bmatrix} \varepsilon_1(t+\Delta t) \\ \eta_1(t+\Delta t) \\ \varepsilon_2(t+\Delta t) \\ \eta_2(t+\Delta t) \\ \vdots \end{bmatrix}
$$

or:

$$ X(t+\Delta t) = \Phi(t)\, X(t) + W(t+\Delta t) $$

where the subscripts denote clocks and $\Phi(t)$ is the state transition matrix.

Let $z_i(t)$ be the phase of clock i at time t relative to the same reference. The observation equations are:

$$\begin{bmatrix} (z_1(t) - z_1(t - \Delta t))/\Delta t \\ (z_2(t) - z_2(t - \Delta t))/\Delta t \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} f_1(t) \\ d_1(t) \\ f_2(t) \\ d_2(t) \\ \vdots \end{bmatrix} + \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \end{bmatrix}$$

or:

$$Z(t,\Delta t) = H(t)\ X(t) + V(t)$$

where $Z(t,\Delta t)$ is the vector of measurements, $H(t)$ is the observation matrix (time-dependent in that the number of weighted clocks with available data may vary), and $V(t)$ is the vector of measurement errors v_i (t).

For uncorrelated parameters, the covariance matrix $Q(\Delta t)$ of the process errors $W(t)$, for sufficiently small Δt, is such that:

$$\frac{dQ}{dt} = \begin{bmatrix} s_{\varepsilon_1}^2 & 0 & 0 & 0 & \cdots \\ 0 & s_{\eta_1}^2 & 0 & 0 & \cdots \\ 0 & 0 & s_{\varepsilon_2}^2 & 0 & \cdots \\ 0 & 0 & 0 & s_{\eta_2}^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

where $s_{\varepsilon_i}^2$ is the noise spectral density (sometimes called variance rate) of random-walk FM and $s_{\eta_i}^2$ is the noise spectral density of random-run FM (random walk of frequency drift). The effect of these errors on the system for any interval Δt is:

$$Q(\Delta t) = \int \Phi(t)\ \frac{dQ}{dt}\ \Phi^T(t)\ dt$$

$$= \begin{bmatrix} s_{\varepsilon_1}^2 \Delta t + s_{\eta_1}^2 (\Delta t)^3/3 & s_{\eta_1}^2 (\Delta t)^2/2 & 0 & 0 & 0 & \cdots \\ s_{\eta_1}^2 (\Delta t)^2/2 & s_{\eta_1}^2 \Delta t & 0 & 0 & 0 & \cdots \\ 0 & 0 & s_{\varepsilon_2}^2 \Delta t + s_{\eta_2}^2 (\Delta t)^3/3 & s_{\eta_2}^2 (\Delta t)^2/2 & 0 & \cdots \\ 0 & 0 & s_{\eta_2}^2 (\Delta t)^2/2 & s_{\eta_2}^2 \Delta t & 0 & \cdots \\ 0 & 0 & 0 & 0 & \ddots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

where Φ^T is the transpose of matrix Φ.

If the measurement errors are uncorrelated, the measurement error vector $V(t + \Delta t)$ has the covariance matrix:

$$R(t) = \begin{bmatrix} s_{v_1}^{\;2} & 0 & 0 & \cdots \\ 0 & s_{v_2}^{\;2} & 0 & \cdots \\ 0 & 0 & \ddots & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

where $s_v^{\;2}$ is the noise spectral density of random walk of phase, or white FM.

The assumption of zero autocorrelation does require prefiltering for such outliers as erroneous phase measurements shared by consecutive first differences. The real data in this and the previous paper were subjected to robust prefiltering.

Rather than noise spectral densities, let us instead solve for the variances $\sigma_v^2 = s_v^2 \Delta t$, $\sigma_\varepsilon^2 = s_\varepsilon^2 \Delta t$, and $\sigma_\eta^2 = s_\eta^2 \Delta t$ as our noise parameters. The Hadamard variance, σ_H, when the noise sources consist only of white noise FM, random-walk FM, and random-run FM, can be expressed as [8]:

$$\sigma_H^{\;2} = \sigma_v^{\;2} \tau^{-1} + (1/6)\, \sigma_\varepsilon^{\;2} \tau + (11/120)\, \sigma_\eta^{\;2} \tau^3 \tag{1}$$

where τ is the sampling time. Our noise parameters may be solved by least squares from this equation of condition. Hadamard variance has the beneficial property of being insensitive to frequency drift.

If the reference were a mean timescale, improved values could be obtained from fits of the individual clocks and recomputed occasionally at times when the performance of a clock appears to have changed significantly.

The first step in the version of the Kalman recursion in which no knowledge of the process noise is required is to calculate the inverse of the transition covariance:

$$P^{-1}(t) = [P^-(t)]^{-1} + H^T(t)\, R^{-1}(t)\, H(t)$$

where the transition covariance matrix predicted for the next step $P^-(t)$ (whose superscript denotes "predicted") can be assumed initially to contain huge covariances.

Second, we compute the Kalman gain K:

$$K(t) = P(t)\, H^T(t)\, R^{-1}(t)$$

where $P(t)$ is the transition covariance matrix of the current step. Next, we update the parameter estimate thusly (where ^ denotes "estimated"):

$$\hat{X}(t) = \hat{X}^-(t) + K(t)\,[Z(t) - H(t)\, \hat{X}^-(t)]$$

Then we make the following predictions for the next step

$$\hat{X}{}^{-}(t + \Delta t) \ = \ \Phi\,(t)\ \hat{X}\,(t)$$

$$P^{-}(t + \Delta t) \ = \ \Phi\,(t)\ P\,(t)\ \Phi^{T}(t) \ + \ Q\,(t)$$

and so on. Each application of the recursion yields an estimate of the system state that is a function of the elapsed time since the last filter update, which can occur any time, i.e. Δt is not necessarily constant, and the data need not be equally spaced. Note that there are two matrix inversions instead of one as in the more common formulation of the Kalman filter. We saw in our previous paper that this is of no consequence **[6]**.

If prefilter analysis of the data indicates the presence of a significant frequency step, the covariances in $P^{-}(t)$ could be reset to huge values in order to reinitialize the filter. As formulated here, the Kalman filter regains the state information within a few steps, and the data are appropriately downweighted until that has occurred. This resetting of the covariances affects only the clock involved, not the entire timescale, which is generated from averaging the Kalman frequencies across the entire ensemble, as explained in the next section.

3 THE KALMAN MEAN

The frequencies and drifts produced by this two-state Kalman filter could be averaged and integrated to generate a Kalman-filter-based mean timescale. The relative sizes of the clock covariances $P^{-}(t)$ would be inversely related to the individual clock weights. Robustness would require the imposition of an upper limit on any one clock's relative weight. If the filter and model operate correctly, the clock variances should slowly converge to steady-state values.

While there is some degradation of the frequency stability of the timescale as a result of the capping of each clock's relative weight, this degradation is rarely significant in practice unless the ensemble is small or very inhomogeneous, neither of which is anticipated to be the case upon implementation. In any case, robustness must be purchased at some price, and this price will have to be decided upon during implementation.

If, in forming the mean timescale, each clock's frequency is weighted according to the inverse of its variance, the timescale would be biased by the "clock-ensemble effect." The prediction error is always too small because the frequency of a clock is correlated with the frequency of the ensemble, since the ensemble includes a contribution from each clock. The weights, which are proportional to the inverse of the variances, are therefore systematically too large, causing a positive feedback that increasingly biases the timescale toward the ensemble's best clock **[9,10]**. This can be avoided if the weight of each clock is based on its stability relative to a Mean of which it is not a part, either by: (1) using another Mean entirely, say one based on cesium clocks rather than masers; or (2) computing a Mean consisting of the rest of the clocks in the ensemble and referring the clock in question to that Mean.   (2) can be accomplished by using the relation:

$$\sigma_{u}{}^{2} = \sigma^{2} / (1 - w)$$

where σ^2 is the uncorrected variance, $\sigma_u{}^2$ is the corrected (unbiased) variance, and weight is the relative weight, assumed to be constant with time **[7]**. In practice, an upper limit would also have to be placed on any one clock's weight.

Parameter and error estimates would be available from the filter in near real time, but practical implementation would require robust detection and rejection of outliers (measurements whose errors are unlikely to have originated in the clock), as well as prompt recognition of time and frequency steps. Kalman filters provide both a forecast of the next data point and an estimate of its uncertainty, so it is possible to implement robust outlier detection, though one must allow for the possibility that discrepant points are indications of a step in time or frequency. Stein proposed an altered gain function to smoothly deweight outliers and an adaptive filter to recognize time and frequency steps **[3]**. Data step detection will be addressed in Section 6.

Initially the filter's knowledge of the parameters will be crude, but in postprocessing one can make use of "future data" by running forward- and backward-moving filters through the data and averaging the corresponding parameters weighted by the number of time steps contributing to each using weights based on filter variances ("smoothing") **[11]**. In order to avoid using the same datum twice, the state at any one time from one of the filters has to be averaged with the predicted state at that time from the other filter. This kind of processing of time data has only been done, to our knowledge, by NIST *a posteriori* for their AT2 timescale **[12]**. This could be done in near real time every time step for some interval of the latest data, though retroactive recomputation would be involved.

4 THE MASERS

The two-state Kalman filter described above, including the forward-and-backward averaging and a correction for the clock-ensemble effect, was applied to 539 days of hourly data selected from those of 12 Sigma Tau/Datum/Symmetricom masers at USNO. The data were produced by our recently renovated Data Acquisition System (DAS), a measurement system involving universal time-interval counters and multiplexed switches. The dominant noise of this system is a phase noise of ~35 ps.

The data were referenced to the Mean of the same 12 masers as computed by the current USNO timescale algorithm **[7]**, which removed constant frequencies and drifts from each steady-state segment of every maser by modelling it against the cesium Mean computed by the same algorithm. In practice, the incestuous nature of the ensemble and its reference can be avoided by referencing the masers to the Mean predicted by the filter and modelling the masers against a Kalman cesium Mean. But in this paper, an independent reference is not necessary, since our objectives are to: (1) test the stability and lack of systematic error of the Kalman Mean relative to the operational Mean for DAS data of both masers and cesiums, rather than the lower-noise maser data investigated in our previous paper **[6]**; (2) determine the time constant of the filter in modelling simulated shocks to the clocks, which will involve only comparisons between shocked and unshocked data; and (3) search for an optimal clock weighting scheme, whose relative values should not change significantly between two strongly correlated references (i.e. an independent cesium Mean and a cesium-calibrated Mean).

The Kalman filter yielded very satisfactory results for the DAS data of each maser. For example, Figure 1 compares the Kalman frequencies with the hourly frequencies of phase for maser NAV12. The 537 days of data shown would have to have been least-squares fit by more than five linear segments in order to be detrended for frequency and drift with our current USNO algorithm **[7]**. Deciding on the placement of such segments requires labor and some subjective judgment that the Kalman filter permits us to avoid.

The Kalman frequencies were not very sensitive to choice of process noise parameters (tuning of the filter), i.e. the fit of each clock's entire Hadamard variance curve by Equation (1) gave sufficiently accurate noise parameters in spite of the occasional small changes in the rate and drift parameters encountered here. The noise parameters for the masers had the following ranges:

$$\sigma_v^2 = 2.0 \cdot 10^{-30} \leftrightarrow 2.3 \cdot 10^{-29}$$

$$\sigma_\varepsilon^2 = 5.8 \cdot 10^{-33} \leftrightarrow 5.5 \cdot 10^{-30}$$

$$\sigma_\eta^2 = 1.2 \cdot 10^{-37} \leftrightarrow 2.3 \cdot 10^{-34}$$

The phase error of the DAS measurement system is ~35 ps, so for an hourly first difference, the observational noise would be $(\sqrt{2} \cdot 35 \text{ ps/hr})^2 = 1.9 \cdot 10^{-28}$.

5 THE KALMAN MASER MEAN

Generation of a mean timescale by the simple integration of the average instantaneous Kalman frequency estimates for the clock ensemble yields a preternaturally stable result reflecting only the evolution of one's clock models. Connection with reality must be maintained through use of the *basic timescale equation* [7,13]:

$$z_t = z_{t-\Delta t} + \sum_i w_t(i) \left\{ x_t(i) - x_{t-\Delta t}(i) + f_t(i)\Delta t - \frac{d_t(i)\Delta t^2}{2} \right\}$$

where z_t is the phase of the reference minus the Mean of the ensemble; $w_t(i)$ is the weight of clock i; $x_t(i)$ is the phase of the reference minus clock i; $f_t(i)$ is the frequency and $d_t(i)$ is the frequency drift of clock i, respectively, relative to the Mean, all at time t; and Δt is the time step. In our case, $\Delta t = 1$ hour. Both the Kalman Mean and its reference, the current USNO Mean, were calculated with this equation; the only difference is that $f_t(i)$ and $d_t(i)$ are the state parameters of the filter and the products of least-squares fits to first differences, respectively. Unless noted otherwise, the individual clock weights are unity unless a data point is rejected in the prefiltering.

One of the computational advantages to averaging clock frequencies (as above) instead of clock phases (as in timescale algorithms that use another form of the basic equation) is the obviation of the time steps in the mean timescale when weighted clocks are added or removed from the ensemble. In the case of the current USNO timescale algorithm, the clock's rate relative to the Mean is determined prior to its addition to the ensemble in order that this rate can be removed by the basic equation, preserving the (assumed) constant frequency of the ensemble. In the case of the Kalman filter under test, however, the clock frequency is determined relative to the current USNO maser Mean, rather than the Kalman Mean being computed, so the rate to be removed is that relative to the USNO maser Mean, to which the Kalman Mean will then be referenced. This will not be the case in an actual implementation, however. In that case, the cesiums will be referenced to the Kalman cesium Mean being computed, and the masers will be calibrated by and referenced to the Kalman cesium Mean.

Averaging the 12 masers together over the time span MJD 52048-52587 (though some of the masers came in as late as MJD 52347), we obtained the resulting Kalman Mean, relative to the USNO maser Mean, is shown in Figure 2. The two Means do not differ more than 1.4 ns from one another. Since both Means have been generated from the same raw data and the clocks in each Mean were ultimately

calibrated against the USNO cesium Mean, they are of very similar long-term stability, and their phases should random-walk away from one another, which indeed appears to be the case.

In order to determine the absolute frequency stabilities of the two sets of maser Means (current and Kalman), we divided the masers into three sub-ensembles of four masers each and performed a three-cornered-hat analysis on each set using the method of Torcaso et al. **[14]**, which corrects for sub-ensemble intercorrelations. The results are given in Figure 3. The Kalman timescale's stabilities are at least as great as those of the current timescale's. Apparently the significant measurement (phase) noise at the shortest sampling times and the flicker FM noise of the masers are not impediments to the use of this filter on masers, though the separation between the two sets of stability curves at the mid-term sampling times in Figure 3 indicates a different apportioning of the process and measuring errors by the two timescale algorithms.

In order to use data efficiently and combine clocks properly in a Kalman mean, we should test possible clock weighting schemes. The criterion for such a scheme would be maximum stability in the sampling time region of most interest for the steering of our Master Clock, which ranges from 1 day (the time constant of our frequency steering) to 25 days (the time constant of our phase steering) **[15]**. It might seem logical to use as clock weights the inverse frequency variances estimated by the Kalman filter. However, it is not possible to investigate weighting schemes using these data, because they were calibrated with a Mean based on the same (equally weighted) clocks, so any stability difference between the Kalman and current Means due to the weighting would only indicate that their respective weighting schemes were different. As we will see in Section 7 for the cesiums, weights based on frequency stabilities for sampling times shorter than the time constant of the filter (found to be 11 days in Section 6) yield better results than those based on longer sampling times. But the case for the masers is complicated by the fact that masers are already beyond the white FM noise regime at our shortest sampling time (1 hour). Indeed, whiteness tests with the MATLAB software package indicate that: (1) the frequency residuals of the individual masers are significantly autocorrelated at sampling times shorter than 16 hours due to each clock's prevailing flicker noise; and (2) the frequency residuals from the Kalman maser Mean are significantly autocorrelated at sampling times shorter than 16 days, probably due to the fact that the masers are being calibrated against an incestuous Mean (one computed from the same data processed by another timescale algorithm) and to the 11-day time constant of the filter.

6 KALMAN FILTER TIME CONSTANT

Before moving on to the noisier data of the cesiums, let us use the maser data to determine the time constant of this Kalman filter when operating on DAS data. This time constant is important because of the effects of process noise, such as that due to environmental shocks to the clocks. Significant shocks on longer timescales than this time constant can be properly handled by the filter in its normal operation, but such shocks that take place on shorter timescales would require special handling, e.g. data rejection if the effect is transient, or clock downweighting and covariance reinitialization if the effect appears to be permanent, at least until the filter can again satisfactorily model the clock's new parameters. Failure to take appropriate action could result in the destabilization of the Master Clock if it were steered to the Kalman-filter-based Mean.

In view of this eventuality, before this filter can be implemented in real time, a test will need to be designed that will recognize frequency excursions in the data that are too rapid to be followed closely by the filter. Perhaps the frequency innovations produced by a Kalman filter tuned for white phase and white FM noises (and, hence, having a much shorter time constant than that of our proposed filter), with any significant frequency divergences taken as indicating a recent frequency step by the clock. Another

possibility, at least for the masers, is to monitor their frequencies on a system with a lower measuring noise than the DAS, e.g. the TSC system used in our previous study **[6]**; cesiums are not profitably studied by such a system because of their greater process noise exceeds the measuring noise of even the DAS.

Prior to developing such a test, we must determine the time constant of the current filter. This can be done by inserting an artificial step in the first-difference data and observing how long the filter frequencies require to return to steady state. Ramp shocks of various sizes and over intervals from 1 hour to 1 day were inserted in the data for each of the 12 masers. A sample of the results is shown in Figure 4 for four of the masers. The average recovery time was found to be 11 days, which we will take as the time constant of this filter.

Consequently, the filter requires a rate-change detector with a time constant much shorter than 11 days.

7 THE CESIUMS AND THE KALMAN CESIUM MEAN

Having tested our filter on the stabler data of the masers, let us now proceed to the noisier data of the cesium clocks, which must ultimately provide the systematic calibration of all the clocks. As stated earlier, however, in this test we will calibrate the frequencies of the cesiums against those of the masers, whose frequencies were in turn calibrated with those of these very cesiums, so there will be incest not present in the envisioned implementation. Doing so for 498 days of data for 51 cesium clocks, we obtained the following ranges for the process noise parameters:

$$\sigma_v^{\;2} = 3.7 \cdot 10^{-28} \leftrightarrow 1.0 \cdot 10^{-27}$$

$$\sigma_\varepsilon^{\;2} = 2.7 \cdot 10^{-32} \leftrightarrow 1.6 \cdot 10^{-29}$$

$$\sigma_\eta^{\;2} = 4.1 \cdot 10^{-36} \leftrightarrow 9.4 \cdot 10^{-33}$$

The noise model assumed by our filter is valid for the cesiums because the DAS measurement phase noise, even had it been a problem for the masers, is swamped by the white FM process noise of the cesiums.

Assuming equal clock weights for the moment, we computed a Kalman cesium Mean whose differences from the current USNO cesium mean are given in Figure 5 and whose frequency stability relative to the USNO maser Mean is depicted in Figure 6. Figure 5 shows no departure of the two Means greater than 8 ns, about what the means would be expected to depart from one another as the result of random walk FM **[16]**. In Figure 6, we see that the two Means are of comparable stability on the long term. The small decrease in stability on the short term is likely due to the more realistic (as opposed to optimistic) modelling of the white noise of the individual cesiums compared to the least-squares fit of the current USNO algorithm. In Section 5 we also noted effects from the different noise modelling for the masers.

Unlike the masers, the cesiums still display white FM noise for sampling times as long as our 1-hour measurement rate, so it might be possible to increase the stability of the Kalman cesium Mean by weighting the clocks unequally. We have not been able to accomplish this with the current USNO algorithm, partly because the clocks are so similar in frequency stability and partly because any clock displaying a change in its parameters is immediately deweighted until it can be remodelled, a procedure

that tends to make the performance of all weighted clocks appear similar. Still, perhaps the more realistic treatment of the process noises by the Kalman filter may make differentiation between the clocks possible.

The Kalman frequencies possess the dominant clock noise, so it is logical to try to weight the clocks according to the inverse frequency variances estimated every time step by the filter. Weighting the clocks thusly, however, only worsened the stability (see Figure 7), probably because the frequency variances are mostly a measure of the noise prevailing around sampling times similar to the time constant of the filter (11 days), while the relative stabilities of the individual clocks evidently change more rapidly.

Perhaps, then, it might be possible to increase the stability of the Kalman cesium Mean by weighting the clocks according to the inverse Allan variances for sampling times much shorter than 11 days. Consequently, the cesium data were weighted using Allan variances for sampling times of 1 hour, 6 hours, 12 hours, and 1 day. Indeed, the shorter the sampling time, the better the stability became (see Figure 7), but the stability did not quite attain that of our equally weighted results. Consequently, it is not likely that one can improve the stability of this Kalman-filter-based Mean with Allan-variance weights.

CONCLUSION

We have shown that:

- the DAS measurement data for the masers, the same used in the operational USNO maser mean timescale, can also be successfully utilized by our Kalman filter to generate a Mean that is both as stable as the operational maser Mean and as that produced from the less noisy TSC data studied previously **[6]**, at least over sampling times including a few weeks, which is necessary if such a timescale were to replace the current one for purposes of steering a Master Clock.

- DAS measurement data for the cesium clocks, the same used in the operational USNO cesium mean timescale, can also be successfully utilized by our Kalman filter to generate a cesium Mean about as stable as the operational cesium Mean, within the limitations of the incestuous frequency calibration. This capability is necessary if such a timescale were used to replace the current one for purposes of calibrating the frequencies and drifts of the masers in our ensemble.

- The time constant of our filter/data combination is 11 days, meaning that significant shocks to the clocks occurring over intervals shorter than 11 days must be isolated and their effects eliminated from the Mean using a method with a more alacritous step response than that of our filter.

No better weighting scheme was found for the cesiums than that of equality among each.

Before our final objective, real-time implementation of this mean timescale algorithm, can be attempted, we must:

- test and implement a method capable of identifying and allowing for state changes too rapid for this proposed filter to handle;

- test whether a free-running Mean predicted by this algorithm can serve as a sufficiently stable reference, rather than the operational USNO maser Mean used in this paper;

- decide whether averaging the state parameters from the forward- and backward-moving filters is preferable to using those of the forward-moving filter, given that the smoothing by the former is purchased at the cost of phase steps in the real-time Mean.

The advantages of replacing our current timescale algorithm with a Kalman-filter-based one would be a reduction in labor and an increase in the mathematical rigor of the Mean computation.

ACKNOWLEDGMENTS

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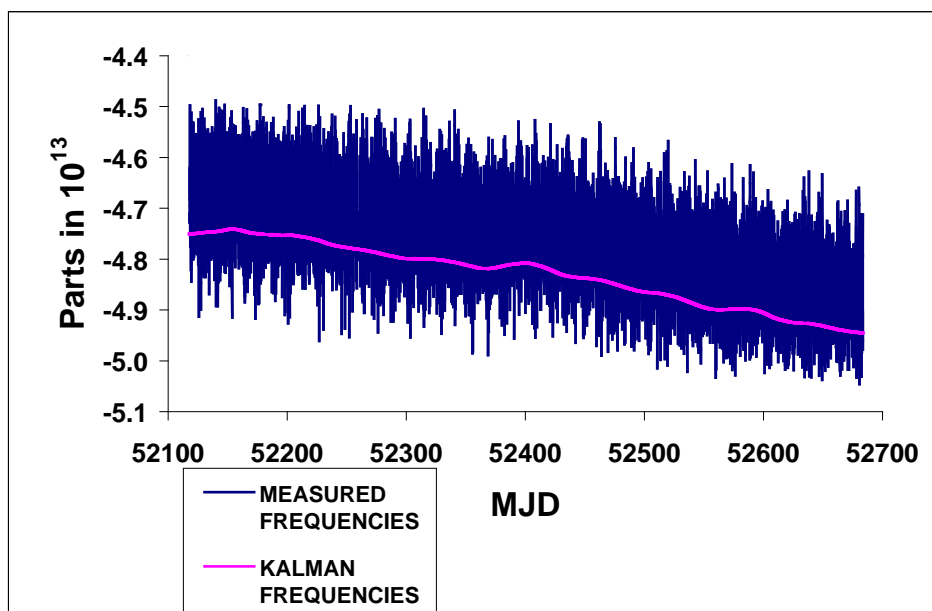
Figure 1. A comparison between hourly frequencies and Kalman frequencies for maser NAV12 relative to the current USNO maser Mean.
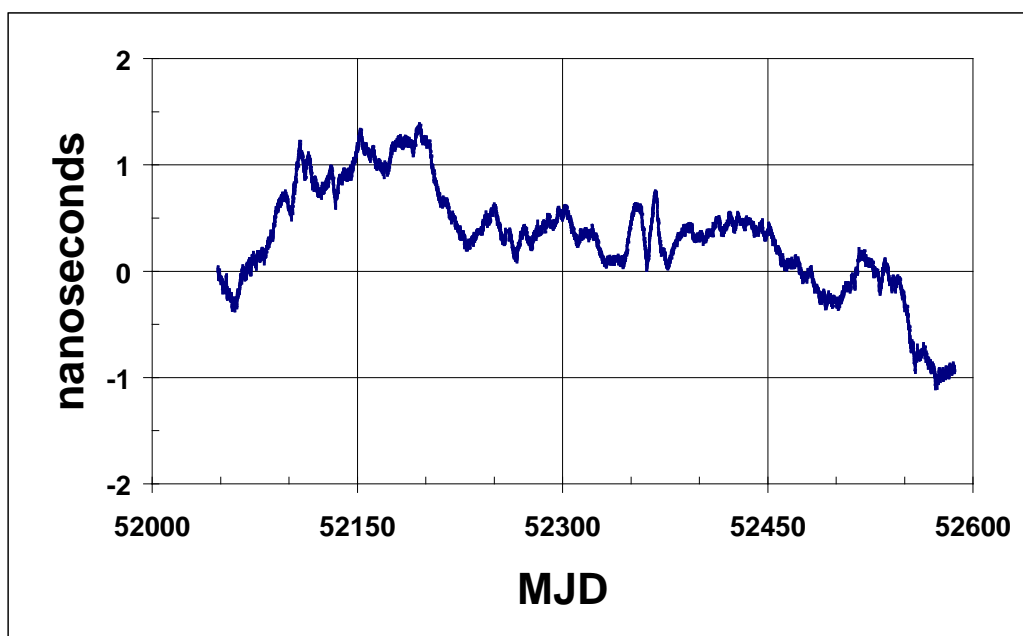
Figure 2. The phase difference between the Kalman maser Mean and the USNO maser Mean. The clocks are equally weighted in both.
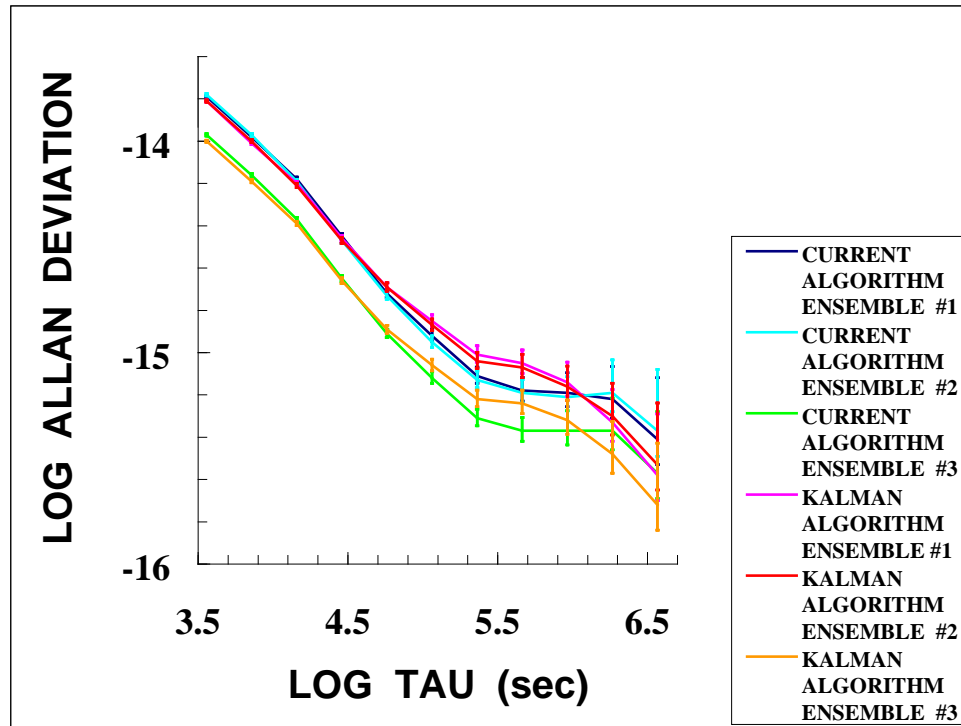
Figure 3. Frequency stability of maser Means for three sub-ensembles as generated by the Kalman filter and the current USNO timescale algorithm. The error bars were computed using the method of Ekstrom and Koppang **[17]**.
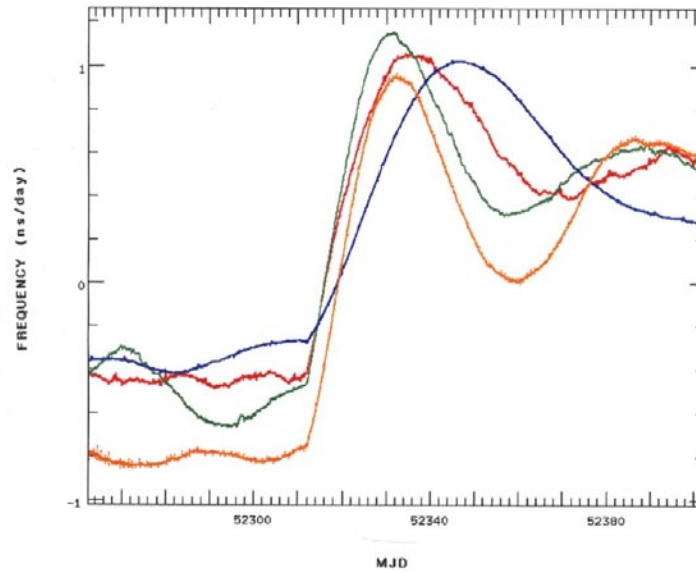
Figure 4. Kalman filter frequencies before and after insertion of a simulated 1 ns/day step at MJD 52312.0 for masers NAV2 (red), NAV4 (green), NAV8 (blue), and NAV11 (orange).
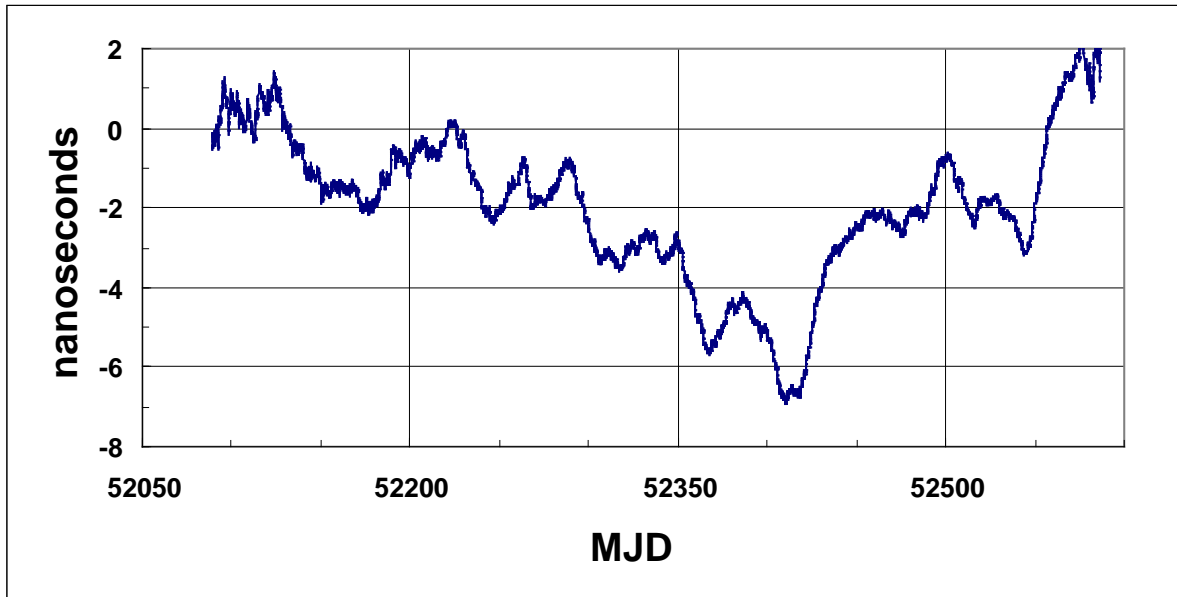

Figure 5. The phase difference between the Kalman cesium Mean and the USNO maser Mean. The clocks are equally weighted in both.
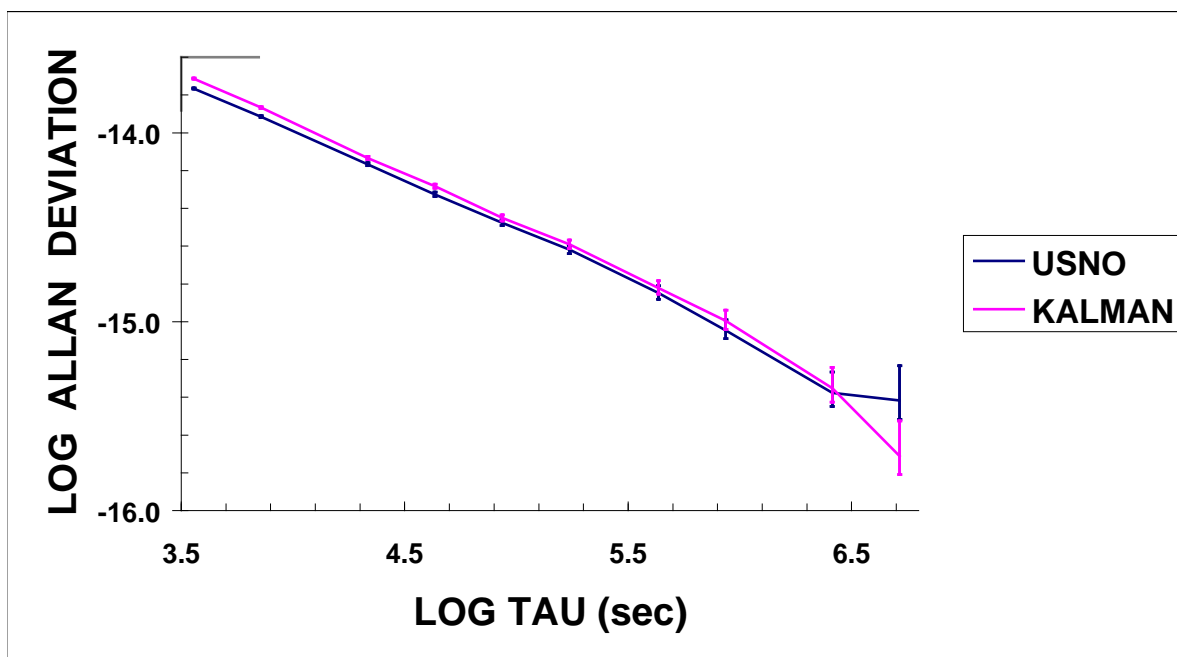
Figure 6. The frequency stability vs. sampling time τ for the Kalman and USNO cesium Means relative to the USNO maser Mean.
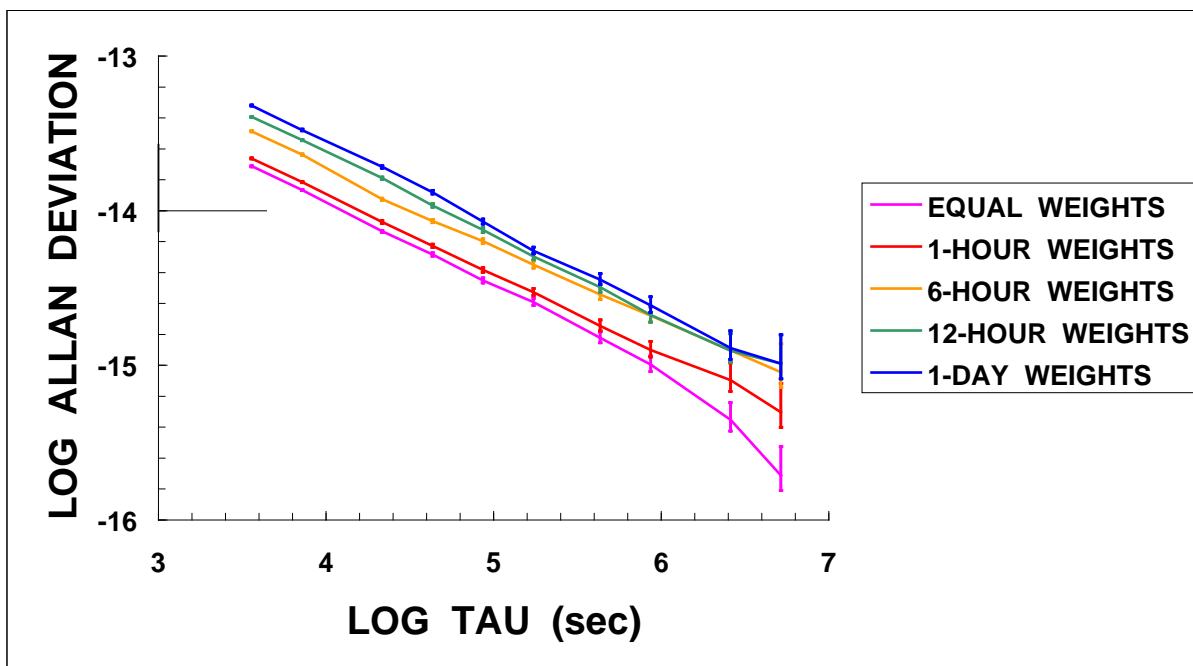

Figure 7. The frequency stability vs. sampling time τ for Kalman cesium Means, relative to the USNO maser Mean, computed for equal clock weights and clock weights based on Allan variances for sampling times of 1 hour, 6 hours, 12 hours, and 1 day.